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A Generalized Reissner-Nordström Solution†

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Abstract

A new class of solutions to the Einstein-Maxwell equations is presented; it stands in the same relation to the Robinson & Robinson (1969) metrics as the Reissner-Nordström solution to Schwarzschild's.

The argument developed in the previous paper (Robinson & Robinson, 1969, hereafter referred to as RR) can easily be extended to a situation in which there is a source-free electromagnetic field. As before, we assume that there exists a vector field k_a which is null, geodetic, shear-free and expanding. Here, in addition, we suppose that k_a is a principal null direction of the electromagnetic field:

$$k_{[a}F_{b]c}k^c = 0$$

From the first condition, it follows that we can construct coordinates subject to equations (2.2) and (2.3).‡ The second condition shows that, in these coordinates,

$$F_{ab} = Vk_{[a}\zeta_{,b]} + \tilde{V}k_{[a}\tilde{\zeta}_{,b]} + (Q + \tilde{Q})\,k_{[a}l_{b]} + P^2(Q - \tilde{Q})\,\zeta_{,[a}\tilde{\zeta}_{,b]}$$

where Q, $\tilde{Q}$, V, $\tilde{V}$ are arbitrary, and

$$l_a\,dx^a := d\rho + Z\,d\zeta + \tilde{Z}\,d\tilde{\zeta} + S\,d\Sigma$$

Substituting into the gravitational equations

$$R_{ab} = 2F_{aq}F^q{}_b - \tfrac{1}{2}F_{pq}F^{qp}g_{ab}$$

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‡ We follow the notation of RR, and cite equations from it by number, without further identification.

we find that five of the six main equations contain no electromagnetic terms. We thus retain (2.1)–(2.12) with only one qualification: S is no longer given by (2.7). Maxwell's equations

$$F^{rs}{}_{;s} = 0, \qquad F_{[rs,\,t]} = 0,$$

reduce to

$$Q = (\rho + i\Omega)^{-2} q, \qquad \tilde{Q} = (\rho - i\Omega)^{-2} \tilde{q}$$
$$V = v - (\rho + i\Omega)\, Q_{/1}, \qquad \tilde{V} = \tilde{v} - (\rho - i\Omega)\, \tilde{Q}_{/2}$$

where invariant derivatives are defined by (3.9), and q, $\tilde{q}$, v, $\tilde{v}$ are functions of ζ, $\tilde{\zeta}$ and σ subject to

$$q_2 + 2\tilde{\Lambda} q = 0, \qquad \tilde{q}_1 + 2\Lambda \tilde{q} = 0,$$
$$v_2 + \tilde{\Lambda} v + [\exp(2u) q]_3 = 0, \qquad \tilde{v}_1 + \Lambda \tilde{v} + [\exp(2u)\tilde{q}]_3 = 0$$

It follows from the last main equation that the squared line-element is

$$ds^2 - (\rho^2 + \Omega^2)^{-1} q\tilde{q}\, d\Sigma^2$$

where ds is defined by equations (2.1)-(2.12). The trivial equation is still satisfied identically; and the subsidiary conditions read:

$$A = \tilde{q} v, \qquad \tilde{A} = q\tilde{v}, \qquad B = \tfrac{1}{2} \exp(-2u)\, v\tilde{v},$$

where A, $\tilde{A}$, B are formed from ds in accordance with (4.3) and (4.4).

In the case

$$v = \tilde{v} = 0,$$

the subsidiary conditions contain no electromagnetic terms, and ds is therefore a vacuum line element. Writing the reduced electromagnetic equations as

$$(\rho^{-2} q)_{/2} = (\rho^{-2} q)_{/3} = (\rho^{-2} \tilde{q})_{/1} = (\rho^{-2} \tilde{q})_{/3} = 0$$

using (B.2), and assuming that $q\tilde{q} \neq 0$, we find that ds satisfies (5.1). This class of line-elements has been examined in RR: to complete the solution, we remark that

$$q = \varepsilon(\zeta) \exp(-2u - 2 \textstyle\int \tilde{L}\, d\tilde{\zeta}), \qquad \tilde{q} = \tilde{\varepsilon}(\tilde{\zeta}) \exp(-2u - 2 \textstyle\int L\, d\zeta),$$

where the functions $\varepsilon(\zeta)$ and $\tilde{\varepsilon}(\tilde{\zeta})$ are disposable. For each of the three cases considered in RR, we may write the last equations more concisely:

$$q = \varepsilon \exp(-2u) I^{1/2}, \qquad \tilde{q} = \tilde{\varepsilon} \exp(-2u) \tilde{I}^{1/2}$$

for (5.10);

$$q = \varepsilon \exp(-2u) J_{,2}, \qquad \tilde{q} = \tilde{\varepsilon} \exp(-2u) \tilde{J}_{,1}$$

for (5.14), (5.15);

$$q = \varepsilon \exp(-2u) \tilde{L}^2, \qquad \tilde{q} = \tilde{\varepsilon} \exp(-2u) L^2$$

for (6.4), (6.7).

The relation of these solutions to the RR metrics is roughly the same as that of the Reissner-Nordström solution to Schwarzschild's. Special cases have previously been given by Robinson & Trautman (1962), Newman, *et al.* (1965) and Debney, *et al.* (1969).

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